UNIVERSITY OF LIVERPOOL

Stability and Control of Smart Grid with Communication Delays

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contents

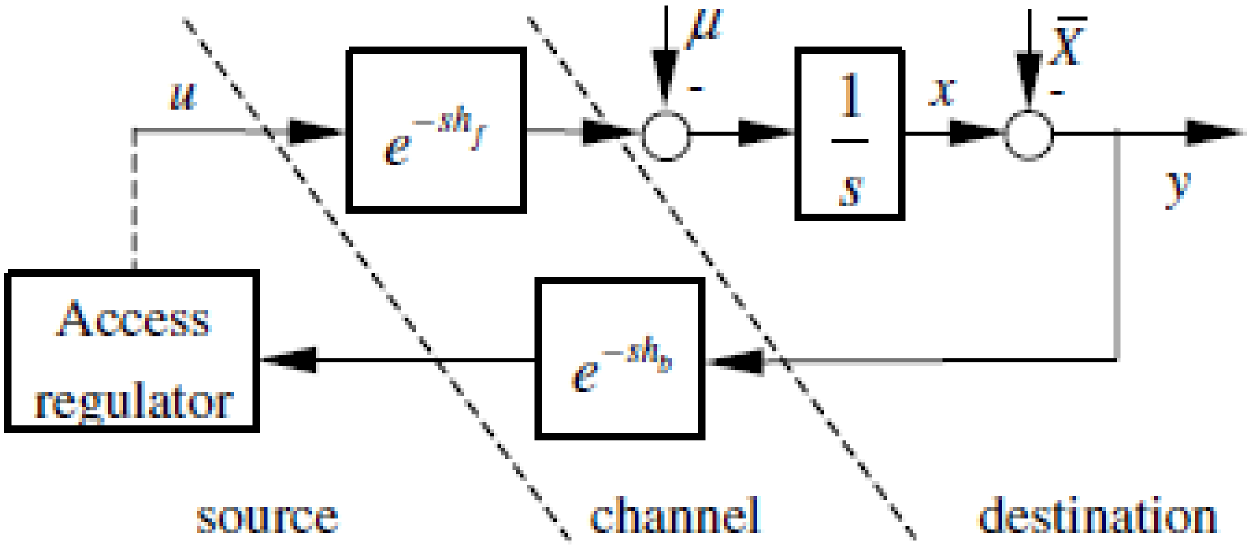
- Stability Analysis of Systems with Time-Varying Delays

Stability Analysis of Systems with Time-Varying Delays

- Background:

In last two decades, communication networks have been among the fastest-growing areas in engineering and there has been increasing interest in controlling systems over communication networks. The control-over-Internet is available because of the high-speed networks.

As the inherent propagation delays of communication networks, these systems are frequently modelled from the control point of view as time-delay systems. In fact, the delays are often time-varying and stochastic which are much more complicated.



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Stability Analysis of Systems with Time-Varying Delays

- Significance:

  Time-delay is inevitable(lead to instability of system)

  The model of systems with time-delay is infinite dimensional functional differential equations

Stability Analysis of Systems with Time-Varying Delays

- Investigation framework:

  Lyapunov-Krasovskii functional(LKF)

  Linear matrix inequality(LMI)

- Target:

  To reduce the conservatism of the obtained criteria(viewpoints of construction of LKFs/estimation of their derivatives)

  Widely apply it in different control systems with time delay

Method

- Choose a LKF candidate, in which Lyapunov matrices are to be determined;
- Find the conditions (C1) that ensure the positive definite of LKF;
- Find the conditions (C2) that ensure the negative definite of LKF’s derivative;
- the summarization of C1 and C2 leads to the stability criterion;

Modeling

- System with time-delay

Investigating the stability problem in time-domain is a usual way to solve it in a system with time-delay

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t - d(t)), & t \geq 0 \\ x(t) = \phi(t), & t \in [-h, 0] \end{cases}$$

where x(t) ∈ R^n is the system state, A and A_d are the system matrices, the initial condition φ(t) is a continuously differentiable function, and d(t) is the time-varying delay satisfying.

LFK Candidate

The delay-product-type LKFs were developed and found to be helpful for improving the results. The selection of The LKF candidates which are more general can give more spaces to find the desired LKF.

$$V_1(t) = \xi_0^T(t)P\xi_0(t)$$

$$V_2(t) = \int_{t-d(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-h}^{t-d(t)} x^T(s)Q_2x(s)ds$$

$$\boxed{V_3(t) = \int_{-h}^{0}\int_{t+s}^{t} \dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha ds}$$

these candidates will be easier to find the feasible solutions due to the freedom of new cross terms

commonly candidates

$$V_4(t) = d_1(t)\xi_1^T(t)P_1\xi_1(t) + d(t)\xi_2^T(t)P_3\xi_2(t)$$

$$V_5(t) = [h_1 - d_1(t)]\xi_3^T(t)P_2\xi_3(t) + [h - d(t)]\xi_4^T(t)P_4\xi_4(t)$$

$$V_6(t) = \int_{t-d(t)}^{t} \begin{bmatrix} x(s) \\ \int_s^t \dot{x}(u)du \end{bmatrix}^T Q_1 \begin{bmatrix} x(s) \\ \int_s^t \dot{x}(u)du \end{bmatrix} ds$$

$$V_7(t) = \int_{t-h}^{t-d(t)} \begin{bmatrix} x(s) \\ \int_{t-d(t)}^s \dot{x}(u)du \end{bmatrix}^T Q_2 \begin{bmatrix} x(s) \\ \int_{t-d(t)}^s \dot{x}(u)du \end{bmatrix} ds$$

Approach for Estimating LKF's Derivative

How to find the boundary of admissible delay, which can remain the system stably, is the significant objective of stability analysis.

double integral term is usually applied in the LKF.

$$\dot{V}_r = \int_{-h}^{0}\int_{t+s}^{t} \dot{x}^T(u)\, R\dot{x}(u)duds$$

find the upper bound of S(t) which can lead to less conservatism on control systems

$$S(t) = -\int_{t-d(t)}^{t} \dot{x}^T(t)R\dot{x}(t)ds - \int_{t-h}^{t-d(t)} \dot{x}^T(t)R\dot{x}(t)ds$$

Approach for Estimating LKF's Derivative

- *Wirtinger-based integral inequality*

$$(b-a)\int_a^b \dot{\omega}^T(s)R\dot{\omega}(s)ds \geq \chi_1^T R\chi_1 + 3\chi_2^T R\chi_2$$

$$\chi_1 = \omega(b) - \omega(a)$$
$$\chi_2 = \omega(b) + \omega(a) - \frac{2}{b-a}\int_a^b \omega(s)ds$$

- Relaxed integral inequalities

$$\begin{bmatrix} M_1R_1^{-1}M_1^T & M_1R_1^{-1}M_2^T & M_1 \\ * & M_2R_1^{-1}M_2^T & M_2 \\ * & * & R_1 \end{bmatrix} \geq 0$$

- Wider integral terms

$$S(t) = -\int_{t-\alpha d(t)}^{t} \dot{x}^T(t)R\dot{x}(t)ds - \int_{t-h}^{t-\alpha d(t)} \dot{x}^T(t)R\dot{x}(t)ds - \int_{t-h}^{t-d(t)} \dot{x}^T(t)R\dot{x}(t)ds$$

Results

- *Wirtinger-based integral inequality*

Criteria	h_1 (s)			h_2 (s)		
	1.0	1.2	1.5	0.3	0.4	0.5
[4]	0.415	0.376	0.248	1.324	1.039	0.806
[5]	0.512	0.406	0.283	1.453	1.214	1.021
[8]	0.519	0.453	0.378	—-	—-	—-
[17]	0.596	0.463	0.313	1.532	1.313	1.140
[7]	0.872	0.672	0.371	1.572	1.472	1.372
Theorem 1	1.163	0.965	0.669	1.875	1.773	1.671

Results

- Relaxed integral inequalities combine Wider integral terms

Criteria	$\mu = -\mu_1 = \mu_2$					
	0	0.1	0.5	0.8	1.0	1000
Corollary 3[11]	2.52	1.81	1.75	1.61	1.60	1.60
Theorem 1 [10]	2.523	2.166	2.028	1.622	1.608	1.608
Theorem 7 [8]	3.034	2.551	2.369	1.700	1.648	1.648
Theorem 1.C1 [7]	3.034	2.551	2.369	1.700	1.648	1.648
Theorem 1.C2 [7]	3.034	2.553	2.373	1.706	1.652	1.652
Theorem 2.C1 [7]	3.136	2.590	2.386	1.775	1.655	1.648
Theorem 2.C2 [7]	3.136	2.598	2.397	1.787	1.665	1.652
Theorem 1	$\mu = -\mu_1 = \mu_2$					
$\alpha = 1$	3.0347	2.4025	1.9075	1.7723	0	0
$\alpha = 0.9$	3.0608	2.4110	1.8929	1.7526	0.0024	0
$\alpha = 0.7$	3.0684	2.4049	1.8721	1.7336	0.0391	0
$\alpha = 0.5$	3.0706	2.4022	1.8602	1.7223	0	0
$\alpha = 0.3$	3.0793	2.4110	1.8709	1.7269	0	0
$\alpha = 0.1$	3.0647	2.4097	1.8903	1.7527	0.0002	0
$\alpha = 0$	3.0345	2.4015	1.9015	1.7604	0.0195	0

Publication

- Xu HT, Zhang CK, Jiang L, and Smith J. Stability analysis of linear systems with two additive time-varying delays via delay-product-type Lyapunov functional. *Applied Mathematical Modelling* 2017;45:955-964.
- Li FD, Xu HT, Zhu Q and Jiang L. A novel relaxed integral inequality and its application to stability analysis of linear systems with time-varying delay has been submitted to Light and Electron Optics in April 2017.

Future work

- the stability analysis of systems (more LKF candidates/different boundary approach)

Thanks